Appendix

In this Appendix we (a) present in more detail the example discussed in the text; and (b) present a simple general argument to show that, under a FOSD shock, knowing the sign of the relationship between price elasticity and income along the individual demand curve says nothing on the relationship between market elasticity and average income.

(a) The example

The income distribution is a standard exponential, with density $f(y,\theta) = e^{-(y-\theta)}$, and cumulative distribution $F(y,\theta) = 1 - e^{-y+\theta}$, $y \in [\theta, \infty)$. As explained in the text (see also f.note 5), $\theta > 0$ is a FOSD parameter and mean income is $\mu(\theta) = 1 + \theta$. We notice that, contrary to our assumption in Proposition 3, $F_\theta(y_m, \theta) = -1 < 0$ and $\pi(y,\theta)$ is independent of θ. Indeed

$$\pi(y,\theta) = 1 + \frac{y f_y(y,\theta)}{f(y,\theta)} = 1 + \frac{-ye^{-(y-\theta)}}{e^{-(y-\theta)}} = 1 - y$$

As to the individual demand function, we have $q(p,y) = \max\left\{1 - \frac{p}{y}, 0\right\}$, so that aggregate demand is

$$Q(p,\theta) = \int_\theta^\infty \max\left\{1 - \frac{p}{y}, 0\right\} e^{-(y-\theta)} dy \tag{A.1}$$

Assume now that $p > \theta$. Then (A.1) becomes

$$Q(p,\theta) = \int_p^\infty \left(1 - \frac{p}{y}\right) e^{-(y-\theta)} dy$$

which gives $Q(p,\theta) = (1 - pA(p)\, e^p) e^{-p+\theta}$ where $A(p) = \int_p^\infty x^{-1} e^x dx$ is a decreasing positive function of p. Clearly, this can be written as $Q(p,\theta) = G(p)e^\theta$ (which is isoelastic in θ), with $G(p) = e^{-p} - pA(p)$.

(b) A simple argument

Assume θ is a FOSD shock to the income distribution, such that $F_\theta(y,\theta) \leq 0$ (strictly somewhere) for all $y \in Y$, which implies $Q_\theta(p,\theta) > 0$ for all $p \in P$. Upon differentiation, a necessary and sufficient condition for $H_\theta(p,\theta) < 0$ is that

$$-pQ_{p\theta}(p,\theta) < Q_\theta(p,\theta) H(p,\theta)$$

where subscripts denote (cross) partials and (obviously) p, $Q_\theta(p,\theta)$, and $H(p,\theta)$ are all positive. We now show that $\eta_y(p,y) < 0$ implies $Q_{p\theta}(p,\theta) < 0$, which means that the LHS is itself positive: some specific assumption on $F(y,\theta)$ is accordingly required beyond FOSD, to ensure that $\eta_y(p,y) < 0$ implies $H_\theta(p,\theta) < 0$.

Integration by parts yields

$$Q_{p\theta}(p,\theta) = -\int_{y_m}^{y_M} q_{py}(p,y)F_\theta(y,\theta)dy$$

Since $\eta_y(p,y) < 0$ implies trivially $-pq_{py}(p,y) < q_y(p,y)\eta(p,y)$ for all y, there follows that

$$pQ_{p\theta}(p,\theta) = -p\int_{y_m}^{y_M} q_{py}(p,y)F_\theta(y,\theta)dy < \int_{y_m}^{y_M} q_y(p,y)\eta(p,y)F_\theta(y,\theta)dy < 0$$

the last inequality deriving from $q_y(p,y)$ and $\eta(p,y)$ being both positive for all y, while $F_\theta(y,\theta) \leq 0$ (strictly somewhere) by the definition of FOSD. Since p is obviously positive, this implies $Q_{p\theta}(p,\theta) < 0$.